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PROBLEMS IN NONLINEAR ACOUSTICS:

SCATTERING OF SOUND BY SOUND, PARAMETRIC ARRAYS, FOCUSED SOUND BEAMS, & NONCOLLINEAR TONE-NOISE INTERACTIONS

Third Annual Summary Report
ONR Contract N00014-85-K-0708
1 July 1988

Mark F. Hamilton

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a REPORT SECURITY CLASSIFICATION: Unclassified

1b RESTRICTIVE MARKINGS:

2a SECURITY CLASSIFICATION AUTHORITY:

2b DECLASSIFICATION / DOWNGRADING SCHEDULE:

3. DISTRIBUTION / AVAILABILITY OF REPORT: Approved for public release; distribution unlimited

4 PERFORMING ORGANIZATION REPORT NUMBER(S): ASX-3

5. MONITORING ORGANIZATION REPORT NUMBER(S):

6a NAME OF PERFORMING ORGANIZATION: The University of Texas at Austin

6b. OFFICE SYMBOL *(If applicable)*:

7a. NAME OF MONITORING ORGANIZATION: Office of Naval Research

6c. ADDRESS *(City, State, and ZIP Code)*:
Department of Mechanical Engineering
The University of Texas at Austin
Austin, TX 78712-1063

7b. ADDRESS *(City, State, and ZIP Code)*:
Physics Division, Code 1112
Arlington, VA 22217-5000

8a. NAME OF FUNDING / SPONSORING ORGANIZATION:

8b. OFFICE SYMBOL *(If applicable)*:

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: N00014-85-K-0708

8c. ADDRESS *(City, State, and ZIP Code)*:

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
61153N	4126943		

11 TITLE *(Include Security Classification)*: Problems in Nonlinear Acoustics: Scattering of Sound by Sound, Parametric Arrays, Focused Sound Beams, and Noncollinear Tone-Noise Interactions

12 PERSONAL AUTHOR(S): Mark F. Hamilton

13a TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Annual Summary	FROM 870701 TO 880630	880701	22

16 SUPPLEMENTARY NOTATION:

17. COSATI CODES

FIELD	GROUP	SUB-GROUP
20	01	

18. SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)*:
nonlinear acoustics; focused sound;
parametric array; acoustic arrays; waveguide;
scattering of sound by sound; noise

19. ABSTRACT *(Continue on reverse if necessary and identify by block number)*

Five projects are discussed involving in this annual summary report, all of which involve basic research in nonlinear acoustics. (1) Scattering of Sound by Sound, a theoretical study of two noncollinear Gaussian beams which interact to produce sum and difference frequency sound. (2) Parametric Receiving Arrays, a theoretical and numerical study of parametric reception in a reverberant environment. (3) Noncollinear Interaction of a Tone with Noise, an experimental investigation of the nonlinear interaction which occurs when the two waves propagate in different modes of a rectangular duct. (4) Nonlinear Effects in Focused Sound Beams, a preliminary analytical study of harmonic generation in the focal region. (5) Parametric Ultrasonic Proximity Sensor, a numerical investigation into the use of a parametric array as an ultrasonic proximity sensor in air. Keywords;

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION: Unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code)	22c. OFFICE SYMBOL
L. E. Hargrove, ONR Physics Division	(202) 696-4221	ONR Code 1112

DD Form 1473, JUN 86 *Previous editions are obsolete.* SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

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INTRODUCTION

This annual summary report describes research performed from 1 July 1987 through 30 June 1988 under ONR Contract N00014-85-K-0708, which began 1 September 1985. The following projects are discussed:

I. Scattering of Sound by Sound

II. Parametric Receiving Arrays

III. Noncollinear Interaction of a Tone with Noise

IV. Nonlinear Effects in Focused Sound Beams

V. Parametric Ultrasonic Proximity Sensor in Air

All five projects involve basic research in nonlinear acoustics. Project I, on the scattering of sound by sound, arose during the investigation of parametric receiving arrays (project II). Project III, on the noncollinear interaction of a tone with noise, received financial support from ONR only from January through May of 1986. This project was completed during the past year. Project IV, which began as an analytical study of focused finite amplitude sound, was short lived. The problem was intended to be the dissertation topic for a new Ph.D. student. However, after having been supported by ONR during October through December of 1987, the student elected to work on another research topic. The leftover salary was used to complete Project V, on the design of a parametric array for use as an ultrasonic proximity sensor in air. Additional information on projects I–IV may be found in the Second Annual Summary Report [1].

The following individuals have been involved with the research effort during the past year.

Senior Personnel

M. F. Hamilton, principal investigator

D. T. Blackstock, resident faculty

J. Naze Tjøtta, visiting scientist (University of Bergen, Norway)

S. Tjøtta, visiting scientist (University of Bergen, Norway)

Neither Blackstock nor the Tjøttas received any financial support under the present contract. Nevertheless, Blackstock (with support from ONR Contract N00014-84-K-0574)

coauthored one of the refereed publications listed below, and the Tjøttas contributed substantially to the overall research effort.

Graduate Students

C. M. Darvennes, Ph.D. student in Mechanical Engineering (projects I and II)

D. J. Driebe, Ph.D. student in Physics (project IV)

Y.-S. Lee, M.S. student in Mechanical Engineering (project V)

S. J. Lind, M.S. student in Architectural Engineering (project III)

The following manuscripts and abstracts, which describe work supported at least in part by ONR Contract N00014-85-K-0708, have been published (or accepted for publication) since 1 July 1988.

Refereed Publications

M. F. Hamilton, J. Naze Tjøtta, and S. Tjøtta, "Noncollinear Interaction of Two Sound Beams from Displaced Gaussian Sources, with Application to Parametric Receiving Arrays," *J. Acoust. Soc. Am.* **82**, 311–318 (1987).

M. F. Hamilton and D. T. Blackstock, "On the Coefficient of Nonlinearity β in Nonlinear Acoustics," *J. Acoust. Soc. Am.* **83**, 74–77 (1988).

M. F. Hamilton and J. A. TenCate, "Finite Amplitude Sound near Cutoff in Higher Order Modes of a Rectangular Duct," *J. Acoust. Soc. Am.* (in press).

T. S. Hart and M. F. Hamilton, "Nonlinear Effects in Focused Sound Beams," *J. Acoust. Soc. Am.* (in press).

Publications in Conference Proceedings

T. S. Hart and M. F. Hamilton, "Nonlinear Effects in Focused Sound Fields," in *Problems of Nonlinear Acoustics, Proceedings of the 11th International Symposium on Nonlinear Acoustics,* edited by V. K. Kedrinskii (Novosibirsk, USSR, August 1987), pp. 192–196.

S. J. Lind and M. F. Hamilton, "Effects of Dispersion on the Nonlinear Interaction of a Tone with Noise in a Duct," *Proceedings of NOISE-CON 88* (Purdue University, Indiana, June 1988), pp. 193–198.

Oral Presentation Abstracts

S. J. Lind and M. F. Hamilton, "Noncollinear Interaction of a Tone with Noise," *J. Acoust. Soc. Am.* **82**, S12 (1987).

C. M. Darvennes and M. F. Hamilton, "Scattering of Sound by Sound from Two Gaussian Beams," *J. Acoust. Soc. Am.* **83**, S4 (1988).

I. SCATTERING OF SOUND BY SOUND

This project arose during a theoretical investigation of parametric receiving arrays operated in reverberant environments (project II). Since July 1987 (the period covered by this report), Darvennes has divided her time equally between the two projects. The work on scattering of sound by sound is being performed in parallel with research begun earlier on the same problem by Naze Tjøtta and Tjøtta, who are currently halfway through a two year sabbatical at Applied Research Laboratories of the University of Texas at Austin (ARL:UT).

A. Background

The problem of the scattering of sound by sound, as well as the terminology, was introduced by Ingard and Pridmore-Brown [2] in 1956. To pose the problem is relatively simple. Two sound beams of different frequencies, radiated from displaced sources, interact at some nonzero angle as shown in Fig. 1. The problem is to calculate or measure the sum or difference frequency sound which is radiated *outside* (i.e., scattered from) the region where the two primary beams intersect. The scattering of sound by sound has received considerable attention over the past three decades (see, e.g., Refs. 3 and 4). During this period, ONR provided support for some of the best known research efforts [5–8]. Whether sound can indeed be scattered by sound is still a hotly contested issue.

Recent theoretical work supports the existence of scattered sound. Specifically, the Tjøttas [9] obtained an asymptotic solution for the scattered sum and difference frequency sound generated by two real beams (e.g., radiation from uniform circular pistons) which intersect at an arbitrary angle. They showed that two different decay rates govern the behavior of the sum and difference frequency sound in the farfield. The portion of the sound field whose directivity function is described by the product $D_1(\theta)D_2(\theta)$, where $D_i(\theta)$ is the

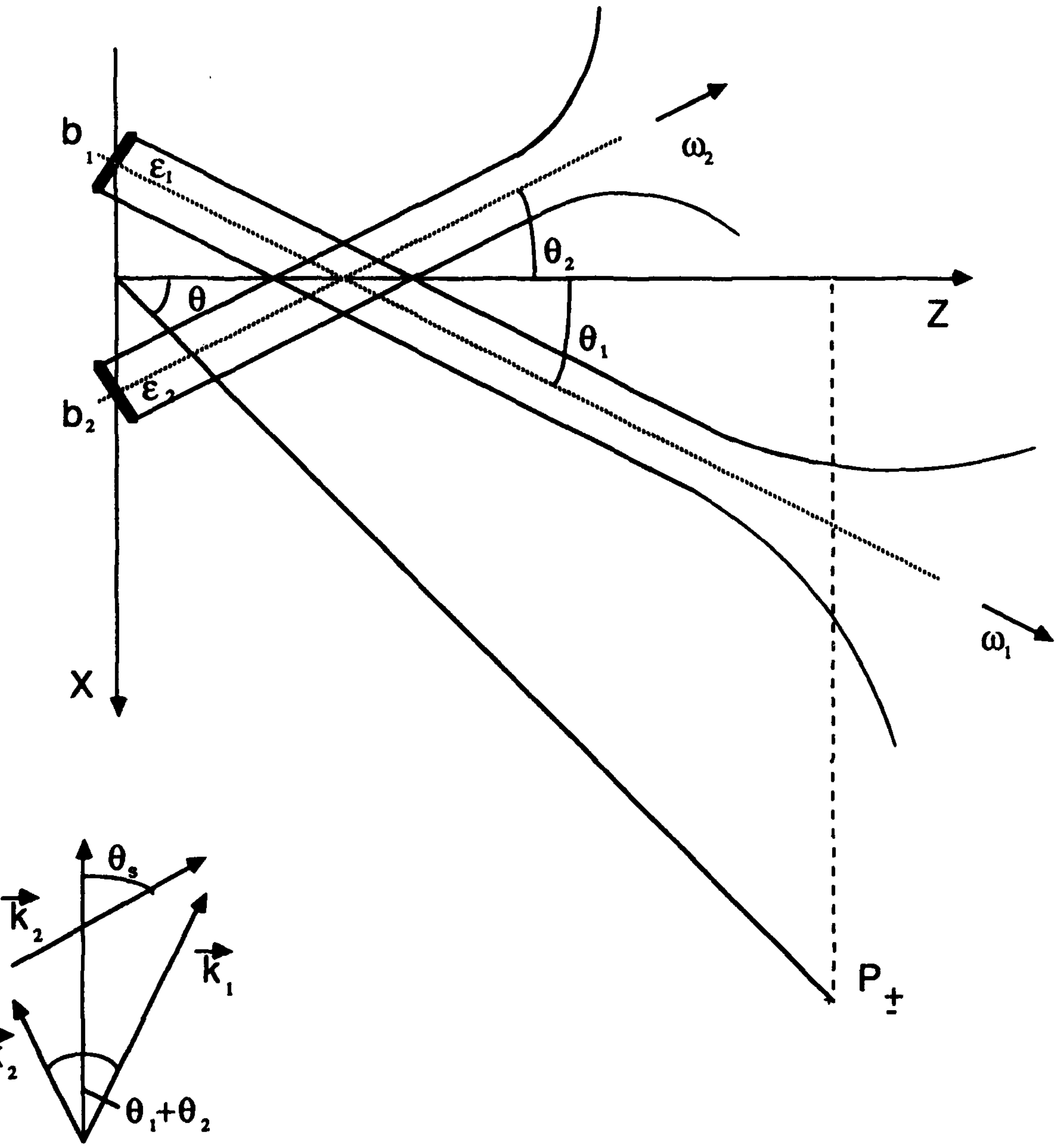

Figure 1

Geometry.

directivity function of the ith primary wave, decays with range z as $\ln z/z$. The scattered sound, whose propagation is governed by a different directivity function $D_s(\theta)$, decays at the faster rate of $1/z$. The Tjøttas have shown that the sum and difference frequency sound can exist in regions where there are nulls in the primary wave fields (e.g., where $D_1(\theta)D_2(\theta) = 0$), and therefore the nonlinearly generated sound satisfies the definition of scattered sound.

B. Results

The results obtained by Darvennes during the past year are summarized by the following abstract of an oral presentation given by Darvennes and Hamilton [10] at the 115th Meeting of the Acoustical Society of America in Seattle, Washington, on May 17, 1988.

> The scattering of sound by sound from Gaussian beams that interact at small angles is investigated theoretically with a quasilinear solution of the Khokhlov–Zabolotskaya nonlinear parabolic wave equation. The analytical solution, which is valid throughout the entire paraxial field, is a generalization of the result obtained for parametric receiving arrays by Hamilton, Naze Tjøtta, and Tjøtta [11]. Significant levels of scattered difference frequency sound are shown to exist outside the nonlinear interaction region. An asymptotic formula reveals that difference frequency sound is scattered in the approximate direction of $\mathbf{k}_1 - \mathbf{k}_2$, where $\mathbf{k}_i$ is the wave vector associated with the direction and frequency of the ith primary beam. Computed propagation curves and beam patterns demonstrate the dependence of the scattered radiation on source separation, frequency ratio, interaction angle, and focusing. Results are also presented for the scattered sum frequency sound. Comparisons are made with the general asymptotic results presented by Berntsen, Naze Tjøtta, and Tjøtta [12], which are valid for arbitrary interaction angles, source separations, and amplitude distributions.

In short, Darvennes' analysis is more limited than that of the Tjøttas because hers is restricted to Gaussian beams which intersect at small angles. In contrast, however, Darvennes' analysis is valid throughout the entire paraxial field, so she can investigate the propagation of scattered sound in the nearfield. The two analyses thus complement each other very well.

We conclude this section with typical results for scattered difference frequency sound from two Gaussian beams. Shown in Fig. 2 is a sound pressure level contour for the difference frequency field generated by two Gaussian beams which intersect at 10°. The primary beams are directed symmetrically about the z axis. The parameter z_0 is the Rayleigh distance of the higher frequency primary beam, and ϵ_0 is the effective source radius of both primary beams. The radiation lobe at the rear of the figure corresponds to the product directivity mentioned above. The other lobe, which exists only beyond $z/z_0 \approx 1$, is the scattered sound, and its direction corresponds approximately with that of $\mathbf{k}_1 - \mathbf{k}_2$.

In Fig. 3 are propagation curves obtained by following the two peaks in the difference frequency sound field. The left column shows a situation where the interaction region is located in the nearfield of one primary beam and in the farfield of the other. In this case the scattered sound (which decays as $1/z$) is dominated at all ranges by the product directivity (which decays as $\ln z/z$). On the other hand, when the interaction region is located in the nearfield of both primary beams (as in the right column), the scattered sound can dominate the product directivity well into the farfield.

The theoretical results obtained by the Tjøttas and Darvennes have motivated experimental work by J. A. TenCate at ARL:UT. The experimental work receives partial support from ONR Contract N00014-84-K-0574 under David T. Blackstock.

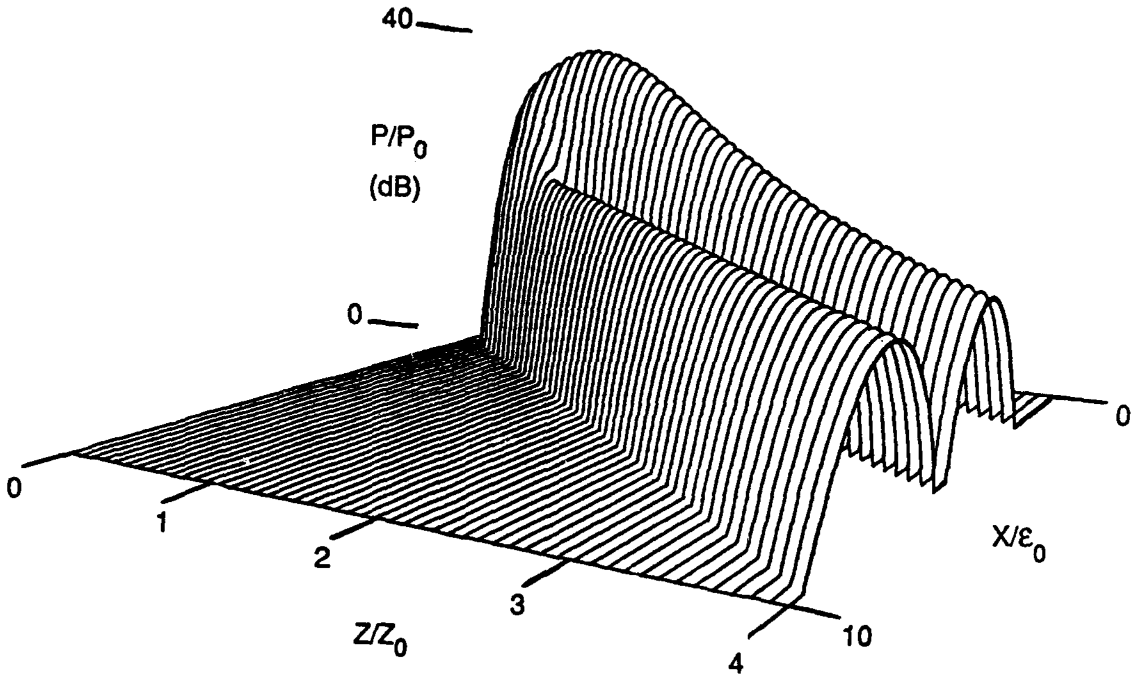

Figure 2

Difference frequency field: $k_2\varepsilon_2 = 30$, $k_1/k_2 = 5$, $b_1 = \varepsilon_1$, and $\theta_1 = 5°$.

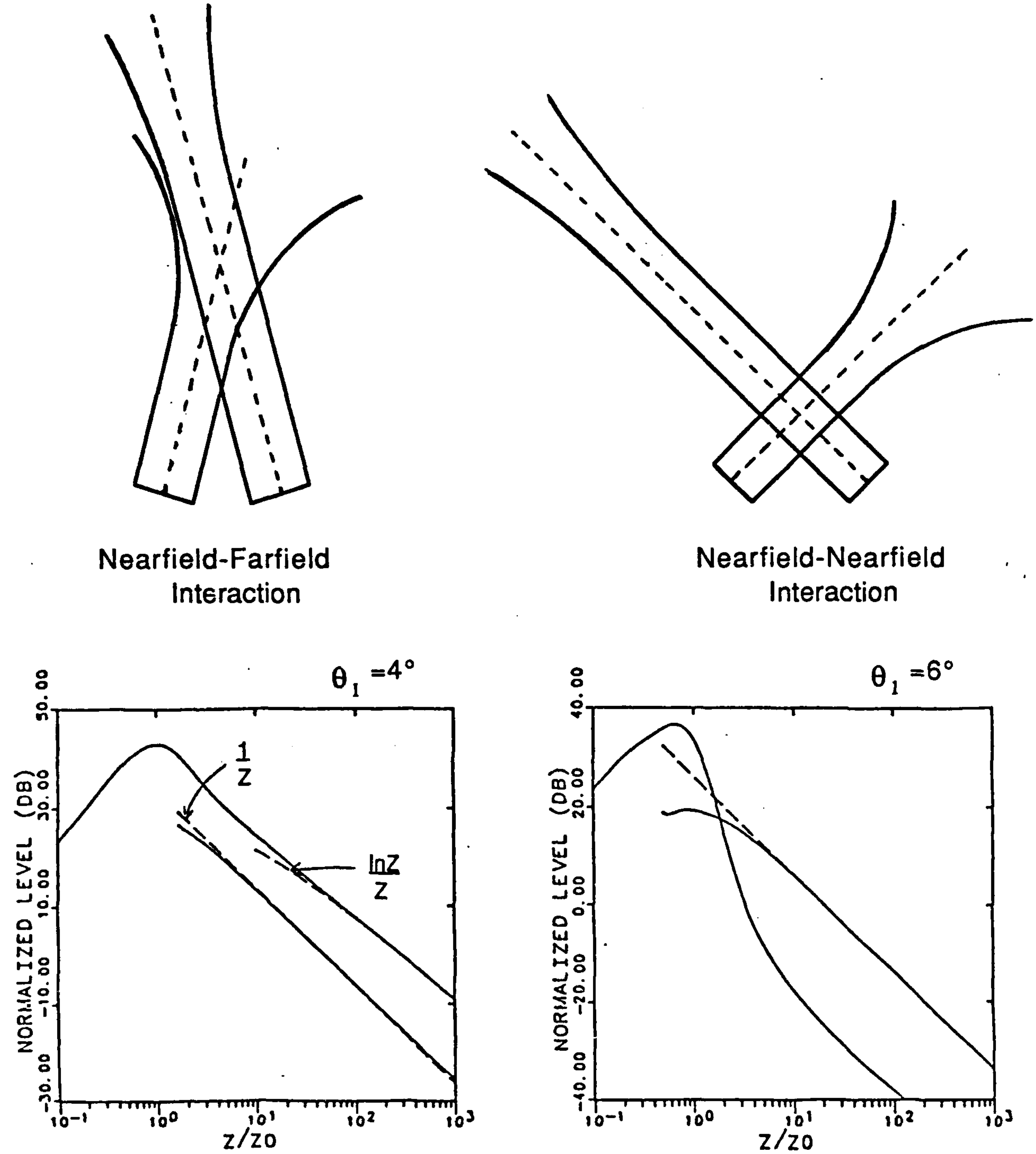

Figure 3

Range dependence of maxima in difference frequency field as θ_1 varies:

$k_2\varepsilon_2 = 30$, $k_1/k_2 = 5$, $b_1 = \varepsilon_1$, and $\theta_1 = 4°, 6°$.

II. PARAMETRIC RECEIVING ARRAYS

The purpose of this research is to investigate the use of a parametric receiving array for directive measurements of noise radiated by ships in reverberant environments. The investigation is theoretical and has been performed by Darvennes since 1 September 1985. The primary objective is to develop a mathematical model that takes into account multipath components from reflecting surfaces. Both theoretical and experimental investigations of related problems are currently underway at ARL:UT [13] and the University of Bergen, Norway [14]. As noted in Sec. I, Darvennes divided her time between this project and the scattering of sound by sound.

A. Background

The motivation for work on parametric reception in reverberant environments was discussed in detail in the Second Annual Summary Report [1] and shall not be repeated here. The problem is posed schematically in Fig. 4. A high frequency pump wave interacts with a low frequency source wave to generate difference frequency sound near a reflecting surface. The goal is to measure the directivity of the source wave by monitoring the nonlinearly generated difference frequency wave. Four of the possible interactions are shown in Fig. 4, namely, those which may involve reflections of both the source wave and the difference frequency wave. Another four interactions arise if reflections of the pump wave are also important. A previous analysis by Hamilton, Naze Tjøtta, and Tjøtta [11] was extended by Darvennes to include not only effects due to the reflecting surface but also absorption. The complete result is a superposition of eight integrals which must, in general, be evaluated numerically.

B. Results

Considerable effort was devoted to assessing the importance of the eight integrals which constitute the entire solution for parametric reception near a reflecting surface in an absorptive medium. It was found that, for most practical situations (with respect to the depth, size, and frequency of the source), the pump wave and the difference frequency wave are sufficiently well collimated that their reflections from the surface may be ignored. However, reflected sound from the source is always important. This leaves only two interactions in Fig. 4, the direct interaction of waves from the pump and the source (top

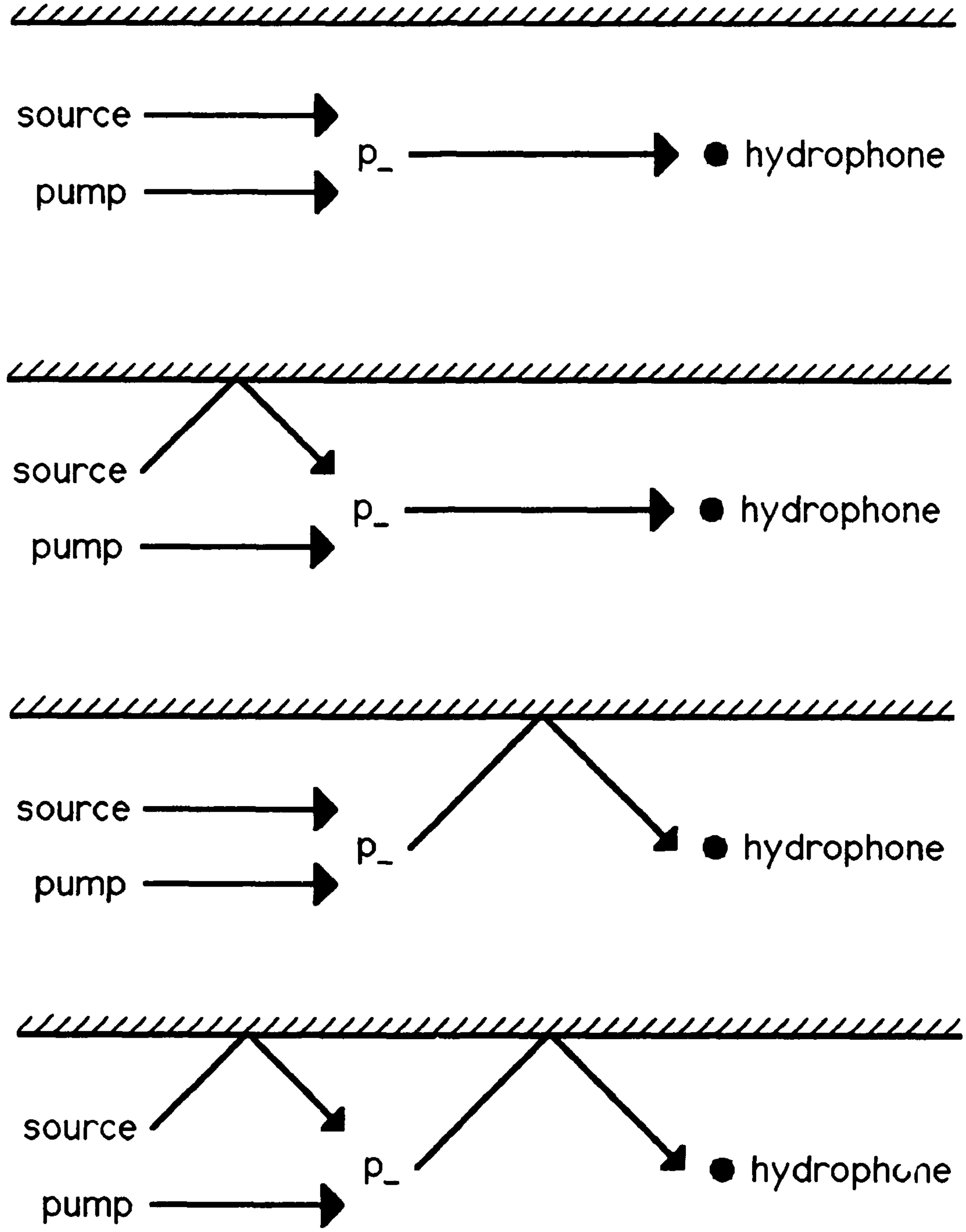

Figure 4

Pump/source wave interactions near a reflecting boundary.

illustration), and the interaction of the pump wave with the reflection of the source wave (second illustration from the top).

A few simple limiting cases (for deep sources is lossless fluids) were discussed in the previous annual summary report [1]. A trade-off was shown to exist in terms of the optimum range for measuring beam patterns with a parametric receiver in multipath environments. When the hydrophone is too close to the source, the beamwidth may be significantly overestimated. On the other hand, multipath components increasingly contaminate the parametrically measured beam pattern as the hydrophone is moved farther away. A source to hydrophone separation of approximately 10 Rayleigh distances was found to provide optimal results for sources located near the reflecting surface.

The effect of absorption on the parametrically measured beam patterns is shown in Fig. 5. The beam patterns are calculated as a function of a dimensionless absorption coefficient a_T (Rayleigh distance divided by absorption length) at a range of 10 Rayleigh distances from a Gaussian source which is submerged at a depth of 2 radii. The solid curves are the parametrically measured beam patterns, the broken curves are the beam patterns which would be measured linearly in a free field environment (i.e, the desired beam patterns), and the dotted curves are the beam patterns which would be measured linearly by a point receiver in the presence of a pressure release reflector. Although the parametrically measured beam patterns are somewhat broader than the desired patterns, they are relatively free of the ripples due to the reflection of the source wave from the surface. Increasing the absorption tends to attenuate the ripples, but at the expense of producing broader parametrically measured beam patterns.

In Fig. 6 are shown parametrically measured beam patterns in a free field environment which demonstrate asymptotic behavior in the presence of absorption. The broken line is the linear beam pattern (which is equivalent to the product directivity), the solid line is the parametrically measured beam pattern, and the dot-dash line is the Westervelt (high absorption) limit. For low absorption the parametrically measured pattern approaches the desired linear directivity, while for high absorption it approaches the Westervelt limit (while becoming increasingly broader than the desired product directivity).

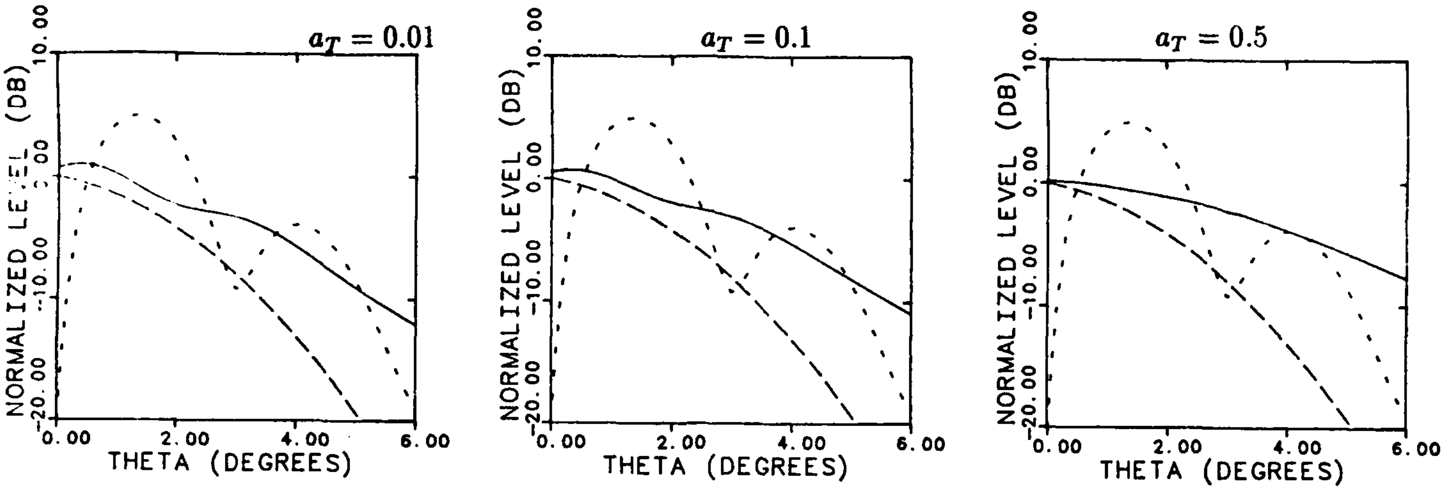

Figure 5

Parametrically (solid curve) and linearly (dotted curve) measured beam patterns near a pressure release surface: $k_2\varepsilon_2 = 30$, $k_1/k_2 = 5$, $\varepsilon_1/\varepsilon_2 = 0.06$, $h = 2\varepsilon_2$, $z = 10k_2\varepsilon_2^2/2$, and $a_T = 0.01, 0.1, 0.5$. The broken curve is the desired free field beam pattern.

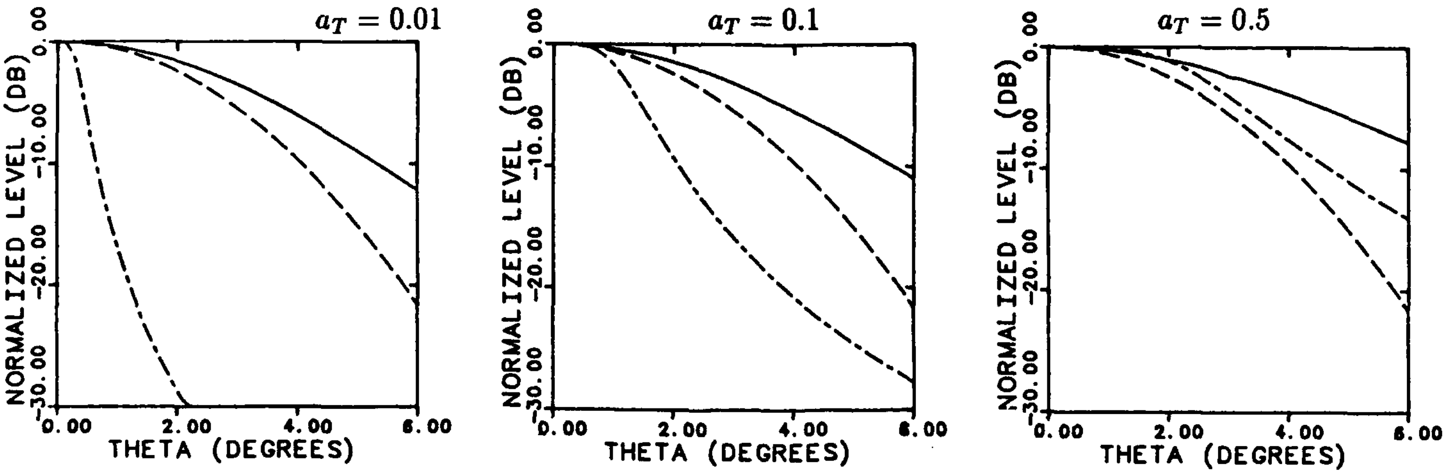

Figure 6

Parametrically measured free field beam patterns (solid curve), compared to product directivity (broken curve) and Westervelt directivity (dot–dash curve): $k_2\varepsilon_2 = 30$, $k_1/k_2 = 5$, $\varepsilon_1/\varepsilon_2 = 0.06$, $h = 2\varepsilon_2$, $z = 10k_2\varepsilon_2^2/2$, and $a_T = 0.01, 0.1, 0.5$.

III. NONCOLLINEAR INTERACTION OF A TONE WITH NOISE

This project received five months of support (January through May, 1986) at the very beginning of the investigation. Virtually all remaining support was provided by the National Science Foundation. The project was carried out by Lind, and it was both theoretical and experimental. It was completed in June 1988, and results from the investigation have been reported by Lind in his master's thesis. Lind is scheduled to receive his M.S. degree in Architectural Engineering in August 1988.

A. Background

The details of this project were reported in the previous annual summary report [1]. Results from the investigation were first presented by Lind and Hamilton [15] at the 114th Meeting of the Acoustical Society of America in Miami, Florida, on November 17, 1987. A written account of the work appeared later in the Proceedings of NOISE-CON 88 [16], which was accompanied by an oral presentation at that meeting on June 20, 1988.

B. Results

A summary of the work is provided by the following abstract from Lind's forthcoming master's thesis.

> The noncollinear interaction of a high-frequency tone with a low-frequency band of noise in a rectangular duct is investigated both theoretically and experimentally. A quasilinear theory developed previously [17] for the noncollinear interaction of two pure tones is generalized to the case where a tone interacts with a band of noise. The spectral shapes of the sum and difference frequency sidebands of noise generated around the high-frequency tone are predicted to vary as a function of range from the source. The variations are due to dispersion which exists when the tone and noise propagate in different waveguide modes with different phase speeds. The sidebands are scalloped in appearance, and the scalloping increases with range from the source. An experiment was performed in which a low-frequency band of noise was transmitted in the (0,0) mode together with a high-frequency tone in the (1,0) mode of an air-filled rectangular duct. The interaction

angle was 55°. Theory is shown to be in good overall agreement with the experimental results.

A sample of theoretical predictions for the frequency spectra due to both noncollinear and collinear interaction is shown in Fig. 7 as a function of range z from the source. The two primary waves consist of a 3000 Hz tone with a sound pressure level of 130 dB (re: 20 μPa) and a noise band from 0 to 500 Hz with a sound pressure level of 100 dB per 10 Hz band. The noncollinear interaction angle is 55°. Except at frequencies which are close to that of the tone, the levels of the sum and difference frequency sidebands are bounded in the case of noncollinear interaction. Experimental results have been obtained which are in very good agreement with the theory.

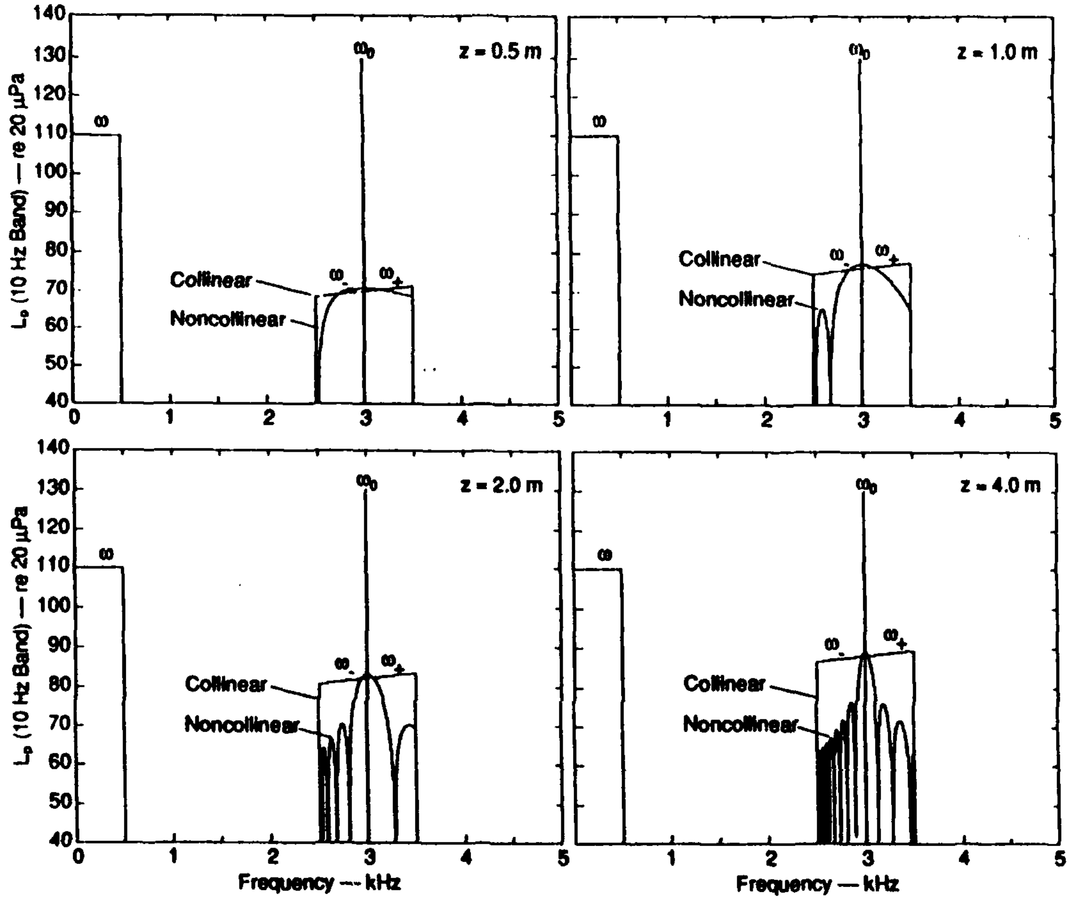

Figure 7

Noncollinear interaction of a 3000 Hz tone with a band of noise from 0 to 500 Hz. The angle of interaction between the two waves is 55°.

IV. NONLINEAR EFFECTS IN FOCUSED SOUND BEAMS

This project was to have been an analytical and experimental extension of the numerical work which was performed by T. S. Hart [18] and described in the previous annual summary report [1]. Driebe, a Ph.D. candidate in physics, worked on the project from February 1 through December 31, 1987. From February 1 through September 30, Driebe's salary was provided by the Independent Research and Development Program at ARL:UT. Because Driebe eventually decided not to work on a topic in acoustics for his dissertation research, financial support for this project was provided by ONR only from October 1 through December 31. The analytical work performed during 1987 was of a very preliminary nature and the experimental work had not yet begun, so there are no new results which warrant presentation at this time.

V. PARAMETRIC ULTRASONIC PROXIMITY SENSOR IN AIR

Funds made available from the termination of project IV (nonlinear effects in focused sound beams) were used to support the completion of numerical work, performed by Lee, on the *design of a parametric array for use as an ultrasonic proximity sensor in air. Work on this* project began during summer 1986. Lee is scheduled to complete his thesis on this topic in time to receive an M.S. degree in Mechanical Engineering in August 1988. Additional support for this project has been provided by the National Science Foundation and the Cray Research Foundation.

A. Background

Ultrasonic proximity sensors are used in robotics and other manufacturing technologies to detect and measure the distance to an object in the same way sonar is used in underwater acoustics. An ideal sensor has (1) long range, (2) high resolution, and (3) small size. However, the absorption of sound by air makes all three characteristics difficult to achieve simultaneously with conventional acoustical designs [19]. Although high resolution can be obtained with a small sensor if sufficiently high frequencies are used, absorption then typically limits the range of operation to less than 1 m. Range can be extended by lowering the frequency, but high resolution then requires a prohibitively large sensor. In underwater applications, the parametric array has been shown to incorporate all three desireable

characteristics. However, the issue of whether a parametric array may provide a viable alternative to conventional ultrasonic proximity sensors in air has not been addressed.

B. Results

When this project began in 1986 it was to involve both experimental and computational work. Because of lack of funding, only the computational portion of the project was continued. The parameters used in the computations were dictated by the following design considerations. The original objective was to build a parametric array with a source having a radius on the order of 1 cm. To avoid difference frequency generation at the source itself, separate (possibly piezoelectric) elements were to be used for generating the two primary waves. It was decided that a source with only two elements would be easiest to build in the laboratory. The source would consist of a circular (disk) element surrounded by a concentric ring element. Computations based on a quasilinear version of the nonlinear parabolic wave equation were thus begun for a bifrequency source consisting of concentric disk and ring elements. Typical operating frequencies for the primary waves were on the order of 200 kHz, which correspond to the frequencies used for operating linear ultrasonic sensors of similar size [19].

Subject to the given design considerations, it was found that the parametric array offers no distinct advantage over conventional ultrasonic sensors used in air. Although narrower beamwidths can be generated parametrically at frequencies as low as 50 kHz, the poor efficiency with which the difference frequency wave is generated prevents the parametric array from achieving greater range than conventional ultrasonic sensors.

Why, then, is the parametric array so successful in underwater applications? First, the background noise level of 80 dB (re: 20 μPa) which is found in typical industrial environments [20] is relatively high with respect to sound pressure levels which can be attained with ultrasonic sensors in air. Sonar systems used in underwater applications operate at source levels which are typically 100 dB above the ambient noise level, whereas ultrasonic sensors in air operate at source levels which may be less than 50 dB above the background noise level. By the time the difference frequency sound from a parametric array in air, because of its lower absorption, outlasts a tone of higher frequency, the levels of both signals are usually below the background noise level of 80 dB. The second reason for the poor performance of parametric arrays in air is the low inherent nonlinearity of

air compared with that of water. Water has a coefficient of nonlinearity which is nearly 3 times that of air, which results in parametric conversion efficiencies which are more than 9 dB higher in water.

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